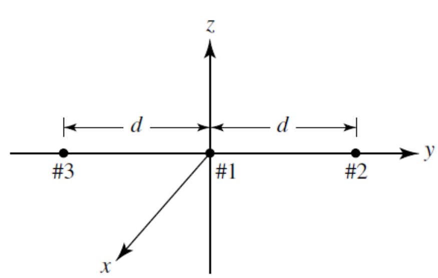
Benha University	**Antennas &Wave Propagation**	**Electrical Eng. Dept.**
Faculty of Engineering	2^{nd}_part	**4^{th} year communication**
Shoubra	-SHEET (2)	**2020-2021**

1. Two very short dipoles ("infinitesimal") of equal length are equidistant from the origin with their centers lying on the *y*-axis, and oriented parallel to the *z*-axis. They are excited with currents of equal amplitude. The current in dipole 1 (at $y = -d/2$) leads the current in dipole 2 (at $y = +d/2$) by 90◦ in phase. The spacing between dipoles is one quarter wavelength. To simplify the notation, let $E0$ equal the maximum magnitude of the far field at distance r due to either source alone.
   (a) Derive expressions for the following six principal-plane patterns:
   i. $|E|$ for $\varphi = 0$◦ ii. $|E|$ for $\varphi = 90$ iii. $|E|$ for $\theta = 90$◦
   (b) Sketch the three field patterns.

2. A very short $(l \le \lambda/50)$ vertical electric dipole is mounted on a pole a height h above the ground, which is assumed to be flat, perfectly conducting, and of infinite extent. The dipole is used as a transmitting antenna in a VHF $(f = 50$ MHz$)$ ground-to-air communication system. In order for the communication system transmitting antenna signal not to interfere with a nearby radio station, it is necessary to place a null in the vertical dipole system pattern at an angle of 80^0 from the vertical. What should the shortest height (in meters) of the dipole be to achieve the desired specifications?

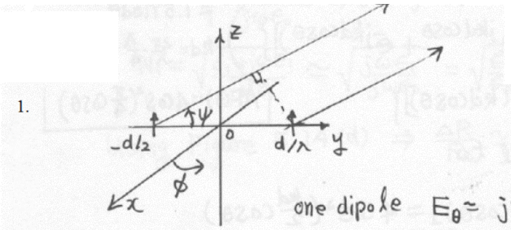
3. It is desired to design an antenna system, which utilizes a vertical infinitesimal dipole of length l placed a height h above a flat, perfect electric conductor of infinite extent. The design specifications require that the pattern of the array factor of the source and its image has only one maximum, and that maximum is pointed at an angle of 60◦ from the vertical. Determine (in wavelengths) the height of the source to achieve this desired design specification. Draw Radiation pattern for AF, infinitesimal dipole and total Radiation pattern

4. Three isotropic elements of equal excitation phase are placed along the *y*-axis, as shown in the figure. If the relative amplitude of #1 is +2 and of #2 and #3 is +1, find a *simplified* expression for the three-dimensional un-normalized array factor.



GOOD LUCK Dr. Gehan Sami

1.



one dipole $E_\theta \simeq j\eta \frac{kI_0 l e^{-jkr}}{4\pi r} \cdot \sin\theta$

Array Factor:

$$(AF)_2 = E_0\left[e^{j\frac{\pi}{2}} \cdot e^{-j\frac{\lambda}{8}\cdot\frac{2\pi}{\lambda}\cos\psi} + e^{+j\frac{\lambda}{8}\frac{2\pi}{\lambda}\cos\psi}\right]$$

$$= E_0 e^{j\frac{\pi}{4}}\left[e^{-j\frac{\pi}{4}(\cos\psi-1)} + e^{j\frac{\pi}{4}(\cos\psi-1)}\right]$$

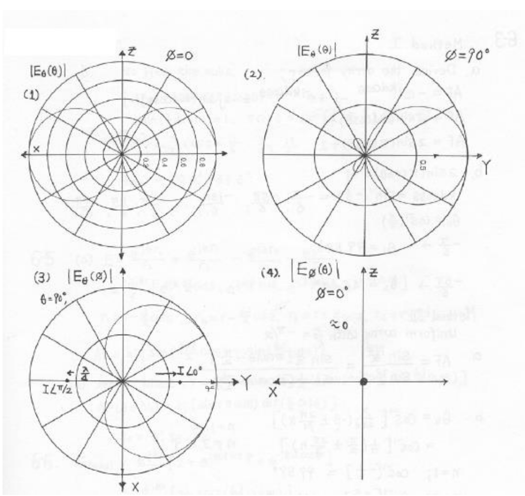
$$= E_0 e^{j\frac{\pi}{4}} \cdot 2 \cdot \cos\left(\frac{\pi}{4}(\cos\psi - 1)\right) = E_0 e^{j\frac{\pi}{4}} \cdot 2\cos\left(\frac{\pi}{4}(\sin\theta\sin\phi - 1)\right)$$

$$\left(\hat{a}_y \cdot \hat{a}_r = \sin\theta \cdot \sin\phi = \cos\psi\right)_{\text{At } y.z \text{ plane}, \phi = 90^\circ} = \sin\theta$$

a. (1) $|E_\theta(\theta)| \propto \left|\sin\theta \cdot \cos\left(\frac{\pi}{4}\right)\right|$, $(x-z$ plane$)$
$\phi = 0^\circ$, $0 \le \theta \le \pi$, $-\pi \le \theta \le \pi$

(2) $|E_\theta(\theta)| \propto \left|\sin\theta \cdot \cos\left(\frac{\pi}{4}(\sin\theta - 1)\right)\right|$, $(y-z$ plane$)$
$\phi = 90^\circ$

(3) $|E_\theta(\phi)| \propto \left|\cos\left(\frac{\pi}{4}(\sin\phi - 1)\right)\right|$, $(x, y.$ plane$)$
$\theta = 90^\circ$



2.

$$E_\theta \sim C_1 \cdot \sin\theta \cdot \cos(kh\cos\theta)\Big|_{\theta=80^\circ} = 0$$

$\cos(kh\cos\theta)\big|_{\theta=80^\circ} = 0$, $\quad kh\cos\theta\big|_{\theta=80^\circ} = \frac{\pi}{2}$, $\quad \frac{2\pi}{\lambda} h\cos\theta\big|_{\theta=80^\circ} = \frac{\pi}{2}$

$$h = \frac{\lambda}{4\cos\theta}\Big|_{\theta=80^\circ} = \frac{\lambda}{4(0.1736)} = \frac{\lambda}{0.6946} = 1.4397\lambda$$

$h = 1.4397\lambda$, $\quad \lambda = \frac{3\times10^8}{50\times10^6} = \frac{30\times10^7}{5\times10^7} = 6$ meters

$h = 1.4397\lambda = 1.4397(6) = 8.6382$ m

$h = 8.6382$ meters

3.

4-43. $E_\theta \simeq j\eta \frac{kI_0 l e^{-jkr}}{4\pi r} \sin\theta \cdot [2\cos(kh\cos\theta)]$

$|AF|_{max} = |\cos(kh\cos\theta)|_{max} = 1$ when $kh\cos\theta_{max} = \pi$

$kh\cos\theta_{max} = \pi$, $kh\cos(60^\circ) = \pi$,

$\frac{2\pi}{\lambda} \cdot h \cdot (\frac{1}{2}) = \pi$, $h = \lambda$

No matter what the height is when $\theta = 90^\circ$, it is a maximum.

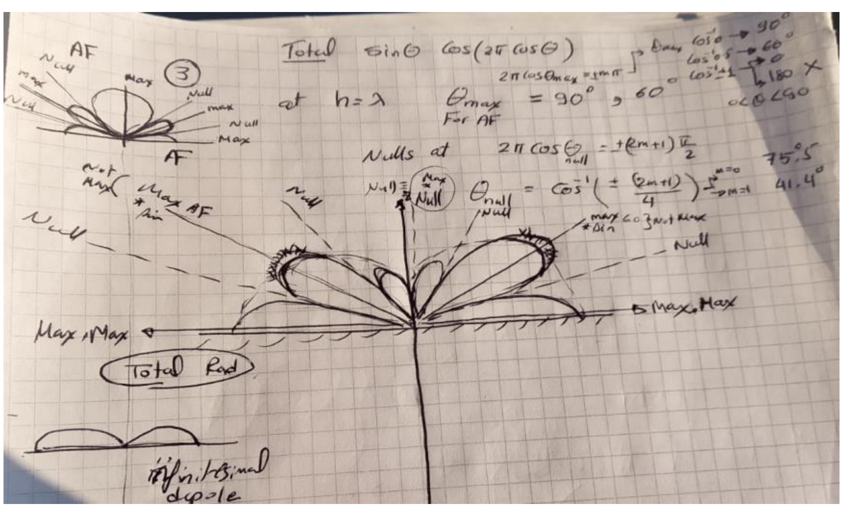
So you always have a maximum at $\theta = 90^\circ$. If you want a maximum at $\theta = 60^\circ$, then $kh\cos\theta = n\pi$, $(n=1,2,3,\cdots)$ leads to a maximum at $\theta = 60^\circ$.

$n=1$: $kh\cos\theta|_{max} = \pi$, $h=\lambda$ leads to maxima at $\theta = 90^\circ, 60^\circ$

If you check closely, it also leads to a maximum at $\theta = 0^\circ$.

So you can not only have one maximum at $\theta = 60^\circ$

Figure 4.15 in lecture for total, so determine nulls and max draw radiation pattern



4.

6-6. $E_{total} = \frac{e^{-jkr}}{r}\left[2 + e^{jkd\cos\psi} + e^{-jkd\cos\psi}\right]$

$= \frac{e^{-jkr}}{r}\left[2 + 2\cos(kd\cos\psi)\right]$

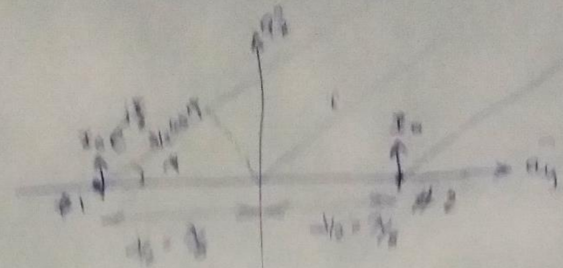
$\cos\psi = \hat{a}_y \cdot \hat{a}_r = \sin\theta \sin\phi$

So, $AF = 2 + 2\cos(kd \sin\theta \sin\phi)$

or $AF = 2[1 + \cos(kd \sin\theta \sin\phi)]$

Sheet 2

(1)



ψ = angle between line of array elements and $\hat{a}_r$

$\therefore \cos\psi = \hat{a}_y \cdot \hat{a}_r = \sin\theta \sin\phi$

for infinitesimal dipole

$E_1 = j\eta \frac{k \ell I_0 e^{j\pi/4}}{4\pi r} e^{-jk(r + \frac{d}{2}\cos\psi)} \sin\theta$, $\frac{kd}{2} = \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = \pi/4$

$E_2 = j\eta \frac{k \ell I_0}{4\pi r} e^{-jk(r - \frac{d}{2}\cos\psi)} \sin\theta$

$\therefore E_{tot} = E_1 + E_2 = j\eta \frac{k \ell I_0}{4\pi r} e^{-jkr} \sin\theta \, e^{j\pi/4} \times 2 \left(\frac{e^{j\pi/4} e^{-j\frac{\pi}{4}\cos\psi} + e^{-j\pi/4} e^{j\frac{\pi}{4}\cos\psi}}{2} \right)$

$= j\eta \frac{k \ell I_0}{4\pi r} e^{-jkr} \sin\theta \, e^{j\pi/4} \times 2 \times \cos\left(\frac{\pi}{4}(\cos\psi - 1)\right)$

$\therefore AF = \cos\left(\frac{\pi}{4}(\sin\theta \sin\phi - 1)\right)$

Element $= \sin\theta$

* Radiation Pattern As Soln manual.

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sheet 6 ③

$\cos(kh\cos 60) = \pm 1 \longrightarrow \cos^{-1}\pm 1 < \begin{matrix} 0 \\ \pi \end{matrix}$

$\therefore \frac{2\pi}{\lambda} \times h \times \frac{1}{2} = \pi$

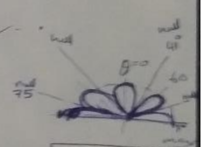
$\therefore \boxed{h = \lambda}$

$\therefore AF = \cos\left(\frac{2\pi}{\lambda} \times \lambda \cos\theta\right)$

$= \cos(2\pi\cos\theta)$

To find max $\longrightarrow$ $2\pi\cos\theta_{max} = \pm n\pi \longrightarrow$ for $n = 0, 1, 2, 3, \ldots$

$\therefore \theta_{max} = \cos^{-1}\frac{\pm n}{2} \longrightarrow n = 0, 1, 2, 3, \ldots$



$0 < \theta < 90$

for $n = 0$ $\quad \theta_{max} = \cos^{-1} 0 = \frac{\pi}{2}$

for $n = 1$ $\quad \theta_{max} = \cos^{-1}\frac{\pm 1}{2} = 60$

for $n = 2$ $\quad \theta_{max} = \cos^{-1} 1 = 0$

nulls at

$2\pi\cos\theta_{null} = \pm(2n+1)\frac{\pi}{2}$

$\therefore \theta_{null} = \cos^{-1} \pm\frac{2n+1}{4} \longrightarrow$ for $n = 0$ $\theta_{null} = \cos^{-1}\frac{1}{4} = 75.5°$

$n = 1$ $\cos^{-1}\pm\frac{3}{4} = 41.4°$

$n = 2$ ×

So for $h = \lambda$ we can not obtain one max at $\theta = 60$

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